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Transient temperature response of different fins to step initial conditions

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Abstract—The transient response of three types of fins (longitudinal, spine and annular), each with three possible shapes (rectangular, triangular and parabolic) is analyzed using numerical solutions. Four initial conditions involving step changes are examined. Two initial conditions concern the heating up of fins by exposing the base to constant temperature or constant heat flux. Two other initial conditions concern the cooling down of operating fins by dissipating heat to the surroundings exclusively, or together with heat dissipation to the base. Characteristics and detailed results from which the temperature distribution at any time can be predicted are presented graphically.

INTRODUCTION

THE TRANSIENT behavior of fins is important in various applications, such as the cooling of electronic components, solar collectors, radiators and compact heat exchangers. Although, in most cases, fins operate in transient conditions, they are usually designed for steady-state operation. Even a car radiator, for example, operates mostly under transient conditions due to the engine load, water temperature and surrounding air velocity that change constantly. For such a case, designing for the worst steady-state conditions is appropriate. For the cooling of electronic components, particularly in aeronautical systems that operate for a short time, or in systems in which weight reduction is vital, the steady-state design would result in overdesign for the operating period. Accurate transient analysis enables one to design fins that would fail in steady-state operation but are sufficient for the desired operating period. Such a design will reduce the total weight significantly.

The performance of fins under steady-state conditions has been studied in considerable detail, but the transient response of such surfaces to changes in either base temperature or base heat flux has not received much attention. Chapman [1] determined the transient response of an annular fin to a step change in the base temperature. The solution obtained by the separation of variables is given in the form of a series. Suryanarayana [2] used the Laplace transform technique which enables him to derive a rapidly convergent approximate solution for the early part of the transient. Aziz and Na [3] used the coordinate perturbation expansion method to obtain the heat transfer rate at the fin base. Chang *et al.* [4] used the variational embedding method to solve the linearized partial differential equation (that was imposed on the governing equation for the steady-state of a straight fin) instead of the original governing equation for the fin's transient response.

The above studies could also be reviewed by taking into consideration the following four categories:

1. Type and shape of the fin.
2. Boundary conditions, especially those that cause the transient response.
3. Method of solving.
4. Nature of results (temperature, heat flux, efficiency, optimization) and their merit.

In the first category, annular fins [1] as well as straight fins [2–4] have been analyzed, but only for constant cross-section shapes. Step changes in the base temperature [1–4] and step changes in the base heat flux, as well as sinusoidal temperature or heat flux [2], were examined. Each of the investigators used different analytical methods that are generally complicated and limited to the simplest shapes. The temperature response [1, 2] and the heat transfer rate [3, 4] were presented for constant thickness fins.

In this study, longitudinal, spine and annular fins, each with rectangular (constant thickness), triangular and parabolic shapes, are examined. Four step changes in the base condition are considered: temperature and heat flux changes, both for heating or cooling the fins. The governing equation is solved numerically which seems to be the only way to relate to all the shapes and conditions. In this study only the temperature response is shown since it is basic to further analysis and optimization.

THE GOVERNING EQUATIONS

Consider one-dimensional conduction in any type of fin—longitudinal, spine or annular of non-constant thickness δ and length L. The heat that enters the

NOMENCLATURE

A_n cross-section area, normal to the heat flow direction [m^2]
B, C variables which distinguish between different fins, Table 1
Bi_L Biot number, $2hL/k$
C_P specific heat [kJ kg^{-1} °C^{-1}]
h heat transfer coefficient [W m^{-2} °C^{-1}]
k thermal conductivity [W m^{-1} °C^{-1}]
L fin length [m]
L_f cut fin length [m]
m_f fin parameter, $L\sqrt{(2h/k\delta_0)}$
n constant for fin shape definition, 0, 1, or 2
P fin perimeter [m]
Q_0 dimensionless base heat rate
q'' base heat flux [W m^{-2}]
R_0 annular fin base radius [m]
T temperature [°C]
T_∞ ambient temperature [°C]
t time [s]
x distance from the fin base and normal to it [m]
Z longitudinal fin width [m].

Greek symbols
α thermal diffusivity [m^2 s^{-1}]
δ fin thickness [m]
θ temperature excess of fin over surroundings [°C]
τ dimensionless time, $\alpha t/L^2$
ϕ normalized temperature, θ/θ_0.

Subscripts
0 fin base
— non-dimensional
a annular
i indicating the location of the element along the fin
j indicating the time step
l longitudinal
s spine
ss steady-state.

fin's base is partially dissipated to the surroundings, assuming constant-average heat convection coefficient, h, and partially heats up the fin (in the instance of heating).

The heat balance for a differential control volume of length dx, assuming no heat is generated within the fin [5], is given by:

$$\frac{\partial}{\partial x}\left(-kA_n\frac{\partial T}{\partial x}\right)+hP\left[\frac{1}{4}\left(\frac{d\delta}{dx}\right)^2+1\right]^{1/2} \times (T-T_\infty)+\rho C_P A_n\frac{\partial T}{\partial t}=0 \quad (1)$$

where x is measured from the fin base and is normal to it, P and A_n are the fin perimeter and area normal to the heat flow at distance x from the fin base, and t is the time elapsed from the step change at the base. The first term results from the conductivity through the cross-section of the fin. The second term results from the convection from the fin surface to the surroundings including the arc length (which is often neglected), and the third term results from the transient change of internal energy of the fin which is reduced to zero for steady-state operation. The convection coefficient, h, is considered to be constant over the fin surface in this study, although it could easily be expanded to be temperature or location dependent.

Introducing the non-dimensional variables: $\bar{A}=A_n/A_0$, $\bar{\delta}=\delta/\delta_0$, $\bar{x}=x/L$, $\phi=(T-T_\infty)/(T_{0,ss}-T_\infty)$, $\alpha=k/\rho C_P$ and $\tau=\alpha t/L^2$ where $T_{0,ss}$ is the base temperature at steady-state (reached eventually for heating and already existing for cooling), into equation (1) and assuming constant conductivity, results in:

$$\frac{\partial^2\phi}{\partial\bar{x}^2}+\frac{1}{\bar{A}}\frac{d\bar{A}}{d\bar{x}}\frac{\partial\phi}{\partial\bar{x}}-\frac{hPL^2}{k\bar{A}A_0} \times\left[\frac{1}{4}\left(\frac{d\bar{\delta}}{d\bar{x}}\right)^2\left(\frac{\delta_0}{L}\right)^2+1\right]^{1/2}\phi-\frac{\partial\phi}{\partial\tau}=0. \quad (2)$$

Three common fin shapes, for the three types of fins, can be defined by a single equation:

$$\bar{\delta}=(1-\bar{x})^n \quad (3)$$

$n=0$ represents the constant thickness fin which has a rectangular shape, $n=1$ describes the triangular fin, and $n=2$ corresponds to the parabolic fin.

The fin perimeter, P, and the normalized area, $\bar{A}$, are defined differently for each of the three types of fins in Table 1. Introducing their definitions and

Table 1. Parameter expressions

Fin	Longitudinal	Spine	Annular
P	$2(Z+\delta)$	$\pi\delta$	$4\pi(R_0+x)$
$\bar{A}_n$	$(1-\bar{x})^n$	$(1-\bar{x})^{2n}$	$\left(1+\frac{\bar{x}}{\bar{R}_0}\right)(1-\bar{x})^n$
B	$\frac{n}{1-\bar{x}}$	$2\frac{n}{1-\bar{x}}$	$\frac{n}{1-\bar{x}}-\frac{1}{\bar{R}_0-\bar{x}}$
C	$1+(1-\bar{x})^n\frac{\delta_0}{Z}$	2	1

derivatives into equation (2) yields a second-order partial differential equation.

$$\frac{\partial^2\phi}{\partial\bar{x}^2}-B\frac{\partial\phi}{\partial\bar{x}}-C\frac{m_f^2}{(1-\bar{x})^n}\times\left[\frac{n^2(1-\bar{x})^{2n-2}}{4}\left(\frac{\delta_0}{L}\right)^2+1\right]^{1/2}\phi-\frac{\partial\phi}{\partial\tau}=0 \quad (4)$$

where the fin parameter $m_f = L\sqrt{((2h)/(k\delta_0))}$ for the three types of fins. Parameters B and C are defined in Table 1 for each type of fin.

In order to solve equation (4) one initial condition and two boundary conditions are needed. In principle, fins could operate by exposing their base to either constant temperature or constant heat flux. The steady-state operation does not distinguish between these two possibilities since one dictates the other. But, for transient operation, when the fin starts to operate, the path to the steady-state temperature distribution differs in the two cases. Therefore, it is of interest to see the transient response of fins to the step connection as well as the disconnection of temperature or heat flux sources to or from the base. Therefore, these four base conditions are examined in this study and will be defined in detail later on.

Case 1. *Sudden exposure to constant temperature*

In this case, initially, the entire fin is maintained at the environmental temperature, T_∞. At time $t = 0$, the fin base is suddenly exposed to a higher temperature, T_0, the temperature increasing with time and with larger time delays at points distant from the base. The temperature rises all over the fin until steady-state distribution is reached. The heat rate entering the fin increases until it is stabilized in steady-state and all the heat entering the fin is dissipated to the surroundings. In this case, $T_{0,ss}$ is easily defined and is kept constant during the process; thus $\phi = 1$ at all times. The boundary and initial conditions are:

$$\phi_{\bar{x}=0} = 1$$
$$\frac{\partial\phi}{\partial\bar{x}} = -Bi_L\phi \quad \text{at} \quad \bar{x} = \bar{L}_f$$
$$\phi(\bar{x},0) = 0. \quad (5a)$$

The second boundary condition is a result of the dissipation of heat at the fin tip to the ambient. This condition will be considered throughout the present study, although other tip conditions could easily be added [6]. L_f is the length from the fin base where the fin terminates. L represents the sharp tip for triangular and parabolic shapes, but for safety it can be cut off at L_f [7].

Case 2. *Sudden exposure to constant heat flux*

In this case, the initial and steady-state temperature distributions are identical to those in the previous case. At time $t = 0$, the fin base is suddenly exposed to a constant heat flux q''_0. The base temperature rises but the slope (heat flux) remains constant. Therefore, $T_{0,ss}$ can be defined only at steady-state with ϕ_0 increasing from 0 to 1. The boundary and initial conditions are:

$$\left.\frac{\partial\phi}{\partial\bar{x}}\right|_{\bar{x}=0} = -\frac{q''_0 L}{k\theta_{0,ss}} = Q_0$$
$$\frac{\partial\phi}{\partial\bar{x}} = -Bi_L\phi \quad \text{at} \quad \bar{x} = \bar{L}_f$$
$$\phi(\bar{x},0) = 0. \quad (5b)$$

Case 3. *Sudden decrease of base temperature*

In this case, the fin is initially at a steady-state temperature distribution. At time $t = 0$, the base temperature suddenly decreases to a temperature lower than $T_{0,ss}$ and the fin cools to a second steady-state distribution. Although any new base temperature can be examined, in this study the base temperature decreases to T_∞. The definition of $T_{0,ss}$ is problematic, considering the new steady-state distribution; therefore, $T_{0,ss}$ for the normalization of ϕ will be considered regarding the initial condition. By decreasing the base temperature, the fin dissipates heat and cools to the surroundings and to the base. The boundary and initial conditions are:

$$\phi_{\bar{x}=0} = 0$$
$$\frac{\partial\phi}{\partial\bar{x}} = -Bi_L\phi \quad \text{at} \quad \bar{x} = \bar{L}_f$$
$$\phi(\bar{x},0) = \phi(\bar{x})_{ss}. \quad (5c)$$

Case 4. *Sudden decrease of base heat flux*

In this case, as in the previous one, the fin is initially at a steady-state temperature distribution. At time $t = 0$, the base heat flux is suddenly decreased to a lower value and the fin cools to a second steady-state distribution. Although any new heat flux can be examined, in this study the base heat flux decreases to zero, which means that no heat interaction takes place between the fin and its base during the cooling process. The difference between this case and the previous one is that in this instance heat is dissipated only to the surroundings. $T_{0,ss}$ is defined in the same way as in the previous case. The boundary and initial conditions are:

$$\left.\frac{\partial\phi}{\partial\bar{x}}\right|_{\bar{x}=0} = 0$$
$$\frac{\partial\phi}{\partial\bar{x}} = -Bi_L\phi \quad \text{at} \quad \bar{x} = \bar{L}_f$$
$$\phi(\bar{x},0) = \phi(\bar{x})_{ss}. \quad (5d)$$

RESULTS AND DISCUSSION

The differential equation was solved numerically by the finite difference method [8] (explicit form) for the three types of fins, each with the three possible shapes. Forward and central differences were used for the time, τ, and distance, $\bar{x}$, respectively, in the numerical method. Equation (4) in its explicit form:

$$\phi_i^{j+1} = \phi_{i+1}^j\left(\frac{\Delta\tau}{\Delta\bar{x}^2} - B\frac{\Delta\tau}{2\Delta\bar{x}}\right) + \phi_i^j\left[1 - 2\frac{\Delta\tau}{\Delta\bar{x}^2} - C\Delta\tau f(\bar{x})\right] + \phi_{i-1}^j\left(\frac{\Delta\tau}{\Delta\bar{x}^2} + B\frac{\Delta\tau}{2\Delta\bar{x}}\right) \quad (6)$$

where i and j indicate the distance and time steps respectively, B and C are shown in Table 1 and $f(\bar{x})$ is defined as:

$$f(\bar{x}) = \frac{m_f^2}{(1-\bar{x})^n}\left[\frac{n^2(1-\bar{x})^{2n-2}}{4}\left(\frac{\delta_0}{L}\right)^2 + 1\right]^{1/2}. \quad (7)$$

Two stability criteria

$$\Delta\bar{x} \leqslant \frac{2}{B} \quad (8)$$

and

$$\frac{\Delta\tau}{\Delta\bar{x}^2} \leqslant \frac{1}{2 + C\Delta\bar{x}^2 f(\bar{x})} \quad (9)$$

are defined from equation (6).

It can be seen from Table 1 that as long as $\bar{R}_0 > 1$, B of the spine is the largest of the three. Introducing B of the spine yields that $\Delta\bar{x}$ should be smaller than $(1-\bar{x})/n$. Keeping in mind that $\bar{x}$ varies from 0 to $1-\Delta\bar{x}$, then the smallest limit for $\Delta\bar{x}$ is $\Delta\bar{x}/2$ for the parabolic ($n = 2$) spine at the tip element. For all other fin shapes, if $n = 0$ or 1 the first stability criterion is satisfied. Notice that at only two elements distant from the tip of the parabolic spine, the criterion is satisfied. Therefore, the parabolic spine was cut at a two element distance from the edge by applying the tip boundary condition at $\bar{L}_f = 1-2\Delta\bar{x}$, without losing the accuracy concerning real fins (that are usually cut). However, $\Delta\bar{x}$ is chosen to be 1/200.

Assuming $\delta_0/L \ll 1$, $f(\bar{x})$ is equal to m_f^2 for $n = 0$, varies from m_f^2 to $m_f^2/\Delta\bar{x}$ for $n = 1$, and varies from m_f^2 to $m_f^2/\Delta\bar{x}^2$ for $n = 2$. Assuming further that $\delta_0/z = 0$ (either very wide or insulated sides of longitudinal fins), then C from Table 1 equals 1 or 2. Because $\Delta\bar{x}$ is very small, the second term of the denominator of equation (9) can be neglected in most cases ($n = 0$ or 2). It cannot be neglected when approaching the tip of the parabolic fin. In this study $\Delta\tau/\Delta\bar{x}^2 = 0.2$ was chosen. This selection leads to unbounded solutions for spines with large fin parameter ($m_f = 5$) at the five elements nearest the tip. The difficulty was overcome by cutting the parabolic fins not too far from the sharp edge, as was described in the previous paragraph.

This solution produced many interesting figures, of which a few are shown here. In each case examined, an effort was made to present the most useful way of obtaining information concerning the transient response of the fins.

Fins are usually designed and chosen on the basis of their efficiency, effectiveness or optimum characteristics. In steady-state operation the design and optimization are well defined. An optimum fin is defined as a fin that dissipates maximum heat by minimum weight. This definition is valid for all design objectives. But, in transient operation the optimization and the design of fins are not obvious, mainly because two heat rates are involved. The heat rate removed from the base and the heat dissipated to the surroundings become equal only in steady-state. Therefore, the design constraint would dictate the optimization procedure. Moreover, for cooling of electronic components the heat rate is dictated and the designer is interested in decreasing the base temperature as much as possible. It is not easy to decide what transient performance is best. The answer could be different for different goals. If the fin is used for removing heat from the base, then the highest amount of heat transferred at a specified time should be maximized. But if the fin is used as a heater then the heat rate convected to the surroundings should be maximized. If the fin is used to reduce the base temperature, then the base temperature should be minimized at a specified time. In any case, this paper presents a first step towards the design and analysis of transient operating fins.

The first case is applicable for systems in which there is a constant base temperature and one is interested in dissipating heat from the base or heating the surroundings. This case is realized in fin–tube heat exchangers, for example, when the fluid in the tubes has a very high heat transfer coefficient. The second case is applicable mostly to cooling of electronic equipment where the heat rate is dictated at the fin base. In this case, one is probably interested in reducing the base temperature. The cooling cases (cases 3 and 4) can be related to two purposes. The first purpose is to define the time required for the fin to reach the ambient temperature when heating takes place in the same way as in the first two cases. The second purpose is related to future analysis of periodical step changes at the base, in which the frequency of the changes is higher than the frequency needed to reach steady-state in heating and in cooling. In this situation, one of the first two cases should be combined with one of the last two cases.

For transient behavior of fins, one of the important parameters would therefore be the time required for a fin to reach steady-state. In the present study, steady-state operation was indicated if the temperature changes were less than 0.1% during $\Delta\tau = 0.01$. An aluminum spine of 10 cm length, for example, will reach steady-state, by the above definition, if the temperature changes were less than 0.1°C during 3 min.

Case 1

In Fig. 1, the normalized temperature distribution development for longitudinal fins with rectangular,

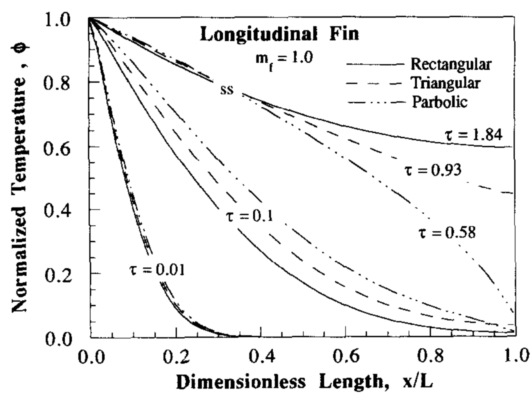

FIG. 1. Normalized temperature of longitudinal fins (case 1).

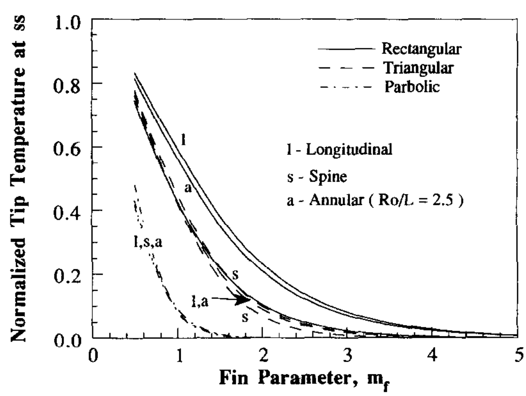

FIG. 3. Normalized tip temperature at steady-state (case 1).

triangular and parabolic shapes is shown. A short time after operating the fin ($\tau = 0.01$), the temperature profiles of all the shapes are almost the same and the heating at the base does not yet affect the fin tip. After further heating, at $\tau = 0.1$, the temperature profile lines of the three shapes are separated, although similar profiles are shown. The steady-state distribution is eventually reached at $\tau = 0.58$ for a parabolic shape, $\tau = 0.93$ (almost twice) for a triangular shape, and at $\tau = 1.84$ (more than three times) for a rectangular one. The steady-state temperature profile of the rectangular fin is concave; for the triangular fin it is approximately linear, but still concave; and for the parabolic fin it is convex.

The same type of behavior has been observed for spines and annular fins. The difference in behavior between the fins is characterized by two parameters: the time required to reach steady-state and the steady-state tip temperature. To enable the user to predict the temperature response, the behavior of both parameters vs the fin parameter are shown in Figs. 2 and 3, respectively.

For all the fins and shapes analyzed in this study, the time required for reaching steady-state decreases as the fin parameter increases, probably due to the increase of the heat convection coefficient. The difference between the steady-state time of different shapes is emphasized for smaller fin parameters. All the fin types show a much faster response for parabolic shapes. This fact will probably dominate the choice of shapes because it reaches full operation faster, and therefore shows a much higher heat dissipation at any time interval during the transient process of heating (as will be discussed in further studies, beyond the scope of this one). The comparison between fin types as shown in Fig. 2 points out that spines are heated faster than longitudinal and annular fins. This statement cannot be comprehensive as long as each fin type has its special parameter, such as R_0/L for annular fins.

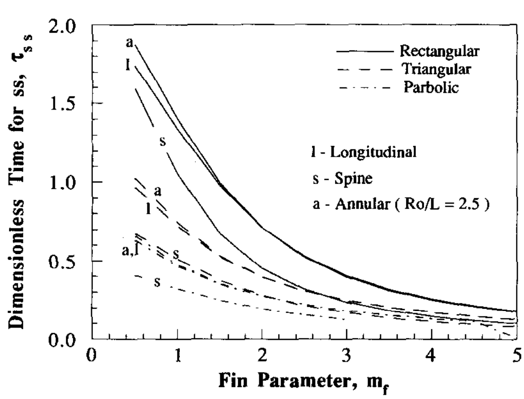

FIG. 2. Dimensionless time required for steady-state (case 1).

The tip steady-state temperatures of all the fins analyzed in this paper are shown in Fig. 3. Once again, the annular fin (for $R_0/L = 2.5$) shows the same behavior as the longitudinal fin. The lowest tip temperatures are observed in spines, especially for rectangular shapes. For longitudinal and annular fins, the tip temperature is highest for rectangular shapes and lowest for parabolic shapes. Only for spines is the tip temperature of the triangular shape higher than that of the rectangular shape at low fin parameters ($m_f < 1.0$). As the fin parameter increases, the tip temperature for all fins decreases until the environmental temperature ($\phi = 0$) is reached. Obviously, for those cases, the fin could be considered as infinitely long [9].

Case 2

In Fig. 4, the normalized temperature distribution advancements for spines with rectangular, triangular and parabolic shapes are shown. The steady-state temperature distribution is much the same as in the previous case. The increase of the base temperature is clearly shown. In this case, as in the previous one, the parabolic-shaped fin requires the least time to reach full operation, although the differences in this case are minor.

The principle behavior of other fin types is essentially the same as for the spine (Fig. 4), only the times required for full operation and the tip temperature are different. In this case, two main parameters influ-

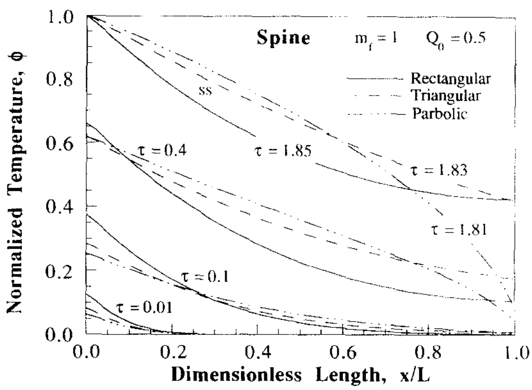

Fig. 4. Normalized temperature of spines (case 2).

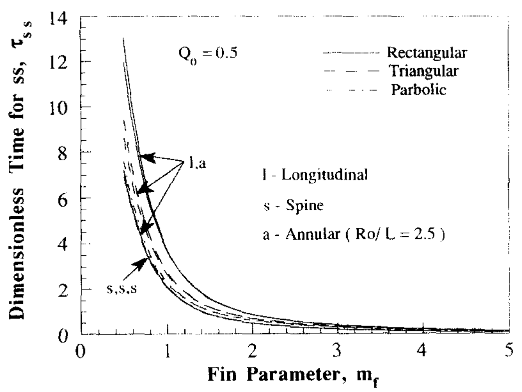

Fig. 5. Dimensionless time required for steady-state vs fin parameter (case 2).

ence the temperature response of the fins: the fin parameter, m_f, and the base heat flux, Q_0.

The dimensionless time required to reach steady-state for all types of fins and shapes are shown in Fig. 5. As in the previous case, the rectangular fin requires the most time to reach steady-state and the parabolic fin the least. Generally, although the steady-state time in this case is much higher than in case 1, the same order of magnitude can be noticed. The effect of the base heat flux, Q_0, on the steady-state time for $m_f = 1$ is shown in Fig. 6. Obviously a higher heat flux requires more time to reach steady-state.

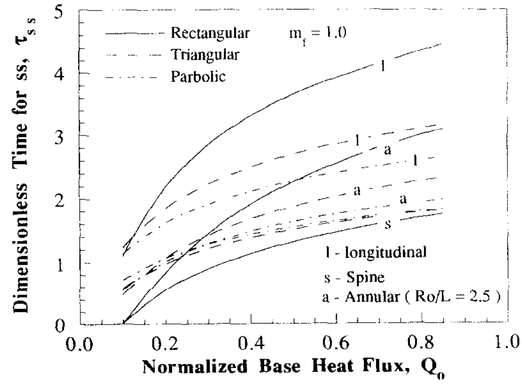

Fig. 6. Dimensionless time required for steady-state vs base heat flux (case 2).

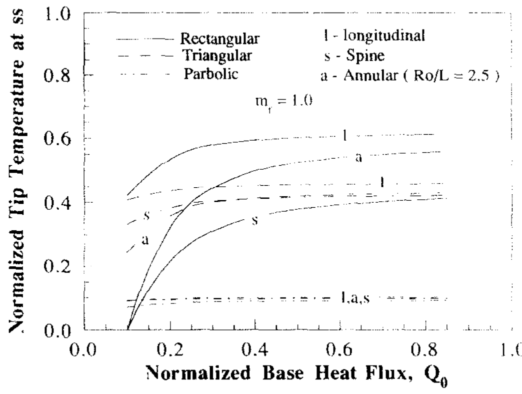

Fig. 7. Normalized tip temperature at steady-state vs base heat flux (case 2).

The normalized tip temperature for $Q_0 = 0.5$ vs the fin parameter, m_f, is almost identical to Fig. 3 of case 1, and therefore is not shown. But, the effect of the base heat flux on the steady-state tip temperature is shown in Fig. 7. For parabolic shapes, the normalized tip temperature is not affected by the base heat flux. For other shapes, the temperature increases and then is stabilized at about $Q_0 = 0.5$.

Case 3

A very detailed description of the normalized temperature reduction of a constant thickness annular fin is shown in Fig. 8. A short time after cooling starts, only the part close to the base experiences temperature changes. Only at $\tau = 0.1$ does the fin tip begin to cool and the direction of heat flow is changed to flow from the tip towards the base. In the case presented in Fig. 8, the whole fin is cooled to the environmental temperature at about $\tau = 1$.

The time required to cool down to the ambient temperature for all the fin types and shapes is shown in Fig. 9. Once again, the parabolic shape response is fastest and the rectangular shape response slowest. The cooling down process of spines requires less time to reach steady-state operation than do other fin types.

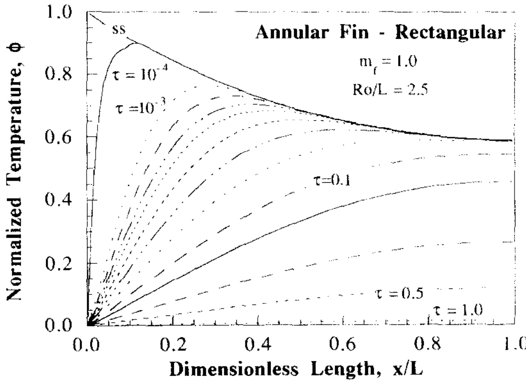

Fig. 8. Normalized temperature of annular fins (case 3).

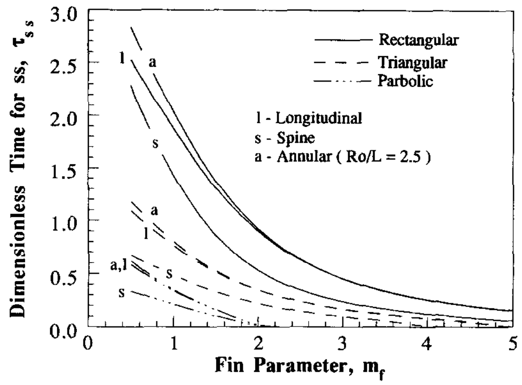

FIG. 9. Dimensionless time required for steady-state (case 3).

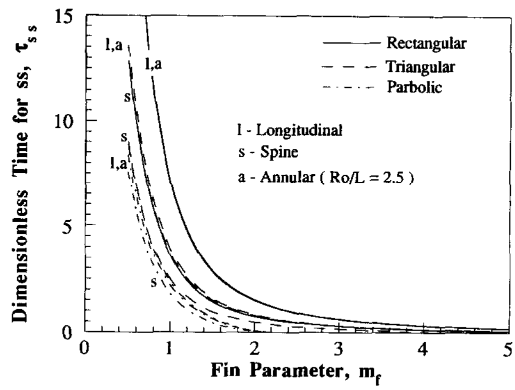

FIG. 11. Dimensionless time required for steady-state (case 4).

Case 4

Figure 10 presents the normalized temperature distribution of longitudinal fins and spines, each with a constant thickness. It can be seen that as the cooling process begins, the base temperature decreases rapidly while the tip temperature remains constant and even rises a bit. Then, a nearly constant temperature is reached all over the fin and the cooling process continues. In this case, the time required for the fin to reach the ambient temperature is longer than in the previous case (the fin dissipates heat only to the ambient). In Fig. 11, as in Fig. 9, the difference between the steady-state time of different shapes is significant—the parabolic fin requires much less time than the rectangular fin.

Although the transient temperature distribution cannot be and is not aimed at addressing questions regarding optimization and selection of fins, by the present analysis some preliminary design rules can be defined. For the same fin type, operation conditions and weight, it is well known that the parabolic shape dissipates more heat than other shapes in steady-state. Combining with the outcome of the present analysis that it also reaches the highest heat in shorter time, then it will certainly dissipate more heat and be preferable for any specified time. In order to confirm this speculation further study should be carried out about the optimal performance for transient behavior. In any case, the present study is a vital step in achieving the final goal.

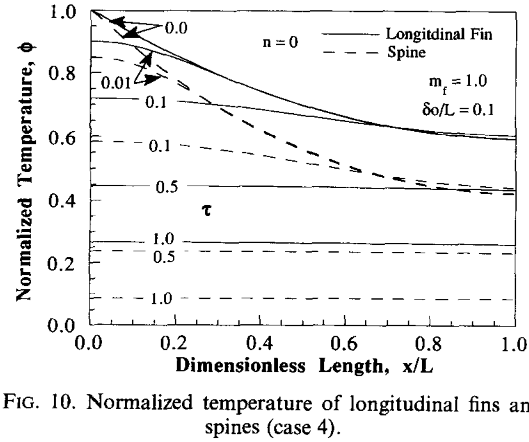

FIG. 10. Normalized temperature of longitudinal fins and spines (case 4).

CONCLUSIONS

The transient response of three types of fins (longitudinal, spine and annular) each with three possible shapes (rectangular, triangular and parabolic) and subject to four cases of heating and cooling were analyzed in this study. The governing equations have been solved numerically to produce figures by which the user is able to predict the temperature distribution along the fin at any time. The time required for the fin to transfer from one defined steady-state to another is presented graphically. In general, more time is needed for a fin to heat up than to cool down. In any case, the parabolic fin requires significantly less time to reach steady-state than do the triangular and certainly the rectangular fins, either to heat up or cool down.

It is well known that parabolic fins have a more optimal shape than do triangular or rectangular ones (i.e. dissipate more heat with the same volume) in steady-state operation. Nevertheless, their use in practice is slight mainly due to manufacturing difficulties and other costs. Their use in transient operation would add one more significant benefit by their ability to reach full operation in much less time, as was shown very clearly in this study.

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